23. x and y are two variables for which 10 pairs of values are available. Further

$$\sum x = 10,\ \sum y = 0,\ \sum x^2 = 148,\ \sum y^2 = 164,\ \sum xy = 123$$

Find the regression co-efficient of y on x.

Solution : The regression co-efficient of y on x

$$b_{yx} = \frac{Cov(x,y)}{{\sigma_x}^2}$$

$$= \frac{\frac{1}{n}\sum(x-\bar{x})(y-\bar{y})}{\frac{1}{n}\sum(x-\bar{x})^2}$$

$$= \frac{n\sum xy-(\sum x)(\sum y)}{n\sum x^2-(\sum x)^2}$$

$$= \frac{10\times123-0}{10\times148-100} = \frac{1230}{1380} = 0.89$$

24. Given that b_{xy} = 0.25, $var(x) = 4,\ var(y) = 36,$ find the correlation between x and y.

Solution : Given, b_{xy} = 0.25

$${\sigma_x}^2 = 4 \quad => \quad \sigma_x = 2$$

$${\sigma_y}^2 = 36 \quad => \quad \sigma_y = 6$$

We know that $b_{xy} = r.\frac{\sigma_x}{\sigma_y}$

⇨ $0.25 = r.\frac{2}{6}$

⇨ $r = 0.75$

25. If r_{xy} = 0.6 and b_{yx} = 0.8, what is the value of b_{xy}?

Solution : We know that

$$r^2 = b_{yx}\ .\ b_{xy}$$

⇨ $(0.6)^2 = 0.8 \text{ x } b_{xy}$

⇨ $\frac{0.36}{0.8} = b_{xy}$

⇨ b_{xy} = 0.45

26. If two regression co-efficients are 0.8 and 1.2, what would be the value of the co-efficient of correlation ?

Solution : We have

$r = \sqrt{b_{yx}b_{xy}}$ = $\sqrt{0.8 \times 1.2}$ = $\sqrt{0.96}$ = 0.98

27. The regression lines have the equations $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$. Find $\bar{x}$ and $\bar{y}$.

Solution : Given the regression lines :

$$x + 2y - 5 = 0$$

$$2x + 3y - 8 = 0$$

Since, both the regression lines passes through the point $(\bar{x}, \bar{y})$, we have

$\bar{x} + 2\bar{y} = 5$ ………………..(i)

$2\bar{x} + 3\bar{y} = 8$ ………………..(ii)

Solving (i) & (ii), we have, $\bar{x}$ = 1 and $\bar{y}$ = 2.

28. Given the following regression line of y on x is $y = 10 - 6x$. Derive the condition under which the regression line of x on y can be written as x = $\frac{1}{6}$(10 – y).

Solution : Given the regression line of y on x

y = 10 – 6x

$\therefore \quad b_{yx} = -6$

If the regression line of y on x be

$$x = \frac{1}{6}(10 - y)$$

⇨ $$x = \frac{10}{6} - \frac{1}{6}y$$

$\therefore \quad b_{yx} = -\frac{1}{6}$

$\text{Now, since both the regression co} - \text{efficients are negative}$

$$r = -\sqrt{(-6)\left(-\frac{1}{6}\right)} = -\sqrt{1} = -1$$

Hence, the required condition is that there must be perfect negative correlation between x and y.

29. Find the line of regression of y on x from the following data :

x :	5	10	15	25	30	35	40	45
y :	25	32	44	32	39	49	55	60

What will be the value of y for x = 48 ?

Solution :

x	y	u = x − 30	v = y − 39	u^2	v^2	uv
5	25	−25	−14	625	196	350
10	32	−20	−7	400	49	140
15	44	−15	5	225	25	−75
25	32	−5	−7	25	49	35
30	39	0	0	0	0	0
35	49	5	10	25	100	50
40	55	10	16	100	256	160
45	60	15	21	225	441	315
		$\sum u = -35$	$\sum v = 24$	$\sum u^2 = 1625$	$\sum v^2 = 1116$	$\sum uv = 975$

Now, we have, $\quad \bar{x} = \bar{u} + 30 = \frac{-35}{8} + 30 = 25.63$

$$\bar{y} = \bar{v} + 39 = \frac{24}{8} + 39 = 42$$

$$\therefore \quad b_{yx} = b_{vu} = \frac{n\sum uv - (\sum u)(\sum v)}{n\sum u^2 - (\sum u)^2} = \frac{8\times975-(-35)\times24}{8\times1625-(-35)^2} = \frac{8640}{11775} = 0.75$$

The regression line of y on x is

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

⇨ $$y - 42 = 0.75(x - 25.63)$$

⇨ $$y = 0.75x + 22.78$$

Since, $x = 48$

∴ $y = 0.75 \times 48 + 22.78$ = 36 + 22.78 = 58.78

30. What is Spearman's Rank Correlation Co-efficient ?

Solution : The Spearman correlation coefficient (r_s) is defined as the Pearson correlation coefficient between the rank variables. For a sample of size n, the n raw scores X_i, Y_i are converted to the ranks $rg_{X_i}, rg_{Y_i}$and it is computed as

$$r_s = \rho_{rg_X, rg_Y} = \frac{cov\,(rg_X,\ rg_Y)}{\sigma_{rg_X}\sigma_{rg_Y}}$$

where,

(i) ρ denotes the usual Pearson Correlation Co-efficient, but applied to the rank variables,

(ii) $cov\ (rg_X, rg_Y)$ is the covariance of the rank variables,

(iii) σ_{rg_X} and σ_{rg_Y} are the standard deviations of the rank variables.

31. What is the formula for Spearman's Rank Correlation Co-efficient ?

Solution : The formula for Spearman's Rank Correlation Co-efficient is

$$\rho = 1 - \frac{\sum_{i=1}^{n} d_i^{\,2}}{2n\sigma_x^{\,2}} = 1 - \frac{6\sum_{i=1}^{n} d_i^{\,2}}{n(n^2-1)}$$

where, n is the number of data points of the two variables and d_i is the difference in the ranks of the ith element of each random variable considered. The Spearman correlation coefficient, ρ can take values from +1 to – 1.