Paper : MTHM – 601 (Statistics)
UNIT – IV : Theoretical Probability Distribution

1. What is discrete uniform distribution ?

Solution : A random variable X or x is said to have a discrete uniform distribution or discrete rectangular distribution over the range $[1, n]$ if its probability mass function (shortly, p.m.f.) is expressed such that

$$P(X = x) = \begin{cases} \frac{1}{n}, for\ x = 1,2, \dots, n \\ 0, \quad\quad otherwise \end{cases}$$

where, n is the parameter of the distribution in the set of all positive integers.

2. What is the moment generating function (shortly, m.g.f.) of discrete uniform distribution with a random variable X ?

Solution : The m.g.f. of X is calculated as follows :

$$M_X(t) = E(e^{tX}) = \frac{1}{n}\sum_{x=1}^{n} e^{tx} = \frac{e^t(1 - e^{nt})}{n(1 - e^t)}$$

3. What is Bernoulli Distribution ?

Solution : A random variable X is said to have a Bernoulli distribution with parameter p if its p.m.f. is given by :

$$P(X = x) = \begin{cases} p^x(1 - p)^{1-x}, for\ x = 1,2, \dots, n \\ 0, \quad\quad otherwise \end{cases}$$

The parameter p lies between 0 and 1, that is, $0 \leq p \leq 1$ and we can write $(1 - p) = q$.

4. Define the Binomial distribution.

Solution : A random variable X is said to have binomial distribution if it assumes positive values and p.m.f. is given by :

$$P(X=x)=p(x)=\begin{cases}\binom{n}{x}p^x q^{n-x}, for\ x=1,2,\dots,n; q=1-p\\ 0, \qquad otherwise\end{cases}$$

5. Ten coins are thrown simultaneously. Find the probability of getting at least seven heads.

Solution : Here, we have,

$p =$ Probability of getting head = $\frac{1}{2}$

$q =$ Probability of getting head = $\frac{1}{2}$

The probability of getting x heads in a random throw of 10 coins is :

$$p(x) = \binom{10}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{10-x} = \binom{10}{x}\left(\frac{1}{2}\right)^{10} ; x = 1,2,\dots,10$$

∴ The probability of getting at least 7 heads is given by :

$$P(X \geq 7) = p(7)+p(8)+p(9)+p(10)$$

$$= \left(\frac{1}{2}\right)^{10}\left\{\binom{10}{7}+\binom{10}{8}+\binom{10}{9}+\binom{10}{10}\right\}$$

$$= \frac{120+45+10+1}{1024} = \frac{176}{1024}$$

6. A and B play a game in which their chances of winning are in the ratio 3:2. Find A's chance of winning at least three games out of five games played.

Solution : Let us consider p is the probability that A wins the game. We are given that $n = 5,\ p = \frac{3}{5}$

∴ $q = 1 - p = \frac{2}{5}$

Hence, by binomial probability law, the probability that out of 5 games played, A wins x games, is given by :

$$P(X=x)=p(x)=\binom{5}{x}\left(\frac{3}{5}\right)^x\left(\frac{2}{5}\right)^{5-x} ; x = 0,1,2,3,4,5$$

∴ The required probability that A wins at least three games is given by :

$$P(X \geq 7) = \sum_{r=3}^{5} \binom{5}{r} \frac{3^r . 2^{5-r}}{5^5}$$

$$= \frac{3^3}{5^5}\left[\binom{5}{3} 2^2 + \binom{5}{4} \times 3 \times 2 + 1 \times 3^2 \times 1\right]$$

$$= \frac{27 \times (40+30+9)}{3125}$$

$$= 0.68$$

7. In each of 4 races, the Democrats have a 60% chance of winning. Assuming that the races are independent of each other, what is the probability that :

(a) The Democrats will win 0 races, 1 race, 2 races, 3 races, or all 4 races ?
(b) The Democrats will win at least 1 race.
(c) The Democrats will win a majority of the races.

Solution : Let us consider, X is equal the number of races the Democrats win.

(a) Using the formula for the binomial distribution, we have,

$$\binom{4}{0} p^0 q^{4-0} = \frac{4!}{0!(4-0)!} \times 60^0 \times 40^4 = 0.40^4 = 0.0256$$

$$\binom{4}{1} p^1 q^{4-1} = \frac{4!}{1!(4-1)!} \times 60^1 \times 40^3 = 4 \times 0.6 \times 0.4^3 = 0.1536$$

$$\binom{4}{2} p^2 q^{4-2} = \frac{4!}{2!(4-2)!} \times 60^2 \times 40^2 = 6 \times 0.6^2 \times 0.4^2 = 0.3456$$

$$\binom{4}{3} p^3 q^{4-3} = \frac{4!}{3!(4-3)!} \times 60^3 \times 40^1 = 4 \times 0.6^3 \times 0.4^1 = 0.3456$$

$$\binom{4}{4} p^4 q^{4-4} = \frac{4!}{4!(4-4)!} \times 60^4 \times 40^0 = 0.6^4 = 0.1296$$

(b) $P\ (at\ least\ 1) = P(X \geq 1) = 1 - P(none) = 1 - P(0) = 0.9744$

Or, $P(1) + P(2) + P(3) + P(4) = 0.9744$

(c) $P(Democrats\ will\ win\ a\ majority)$ = P(X ≥ 3) = P(3) + P(4) = 0.3456 + 0.1296 = 0.4752