6. **Question :** Calculate by Trapezoidal Rule and Simpson's One-Third Rule, Simpson's Three-Eighth Rule an approximate value of $\int_{-2}^{2} x^4\,dx$ with five equidistant ordinates.

Solution : Suppose $\mathrm{I} = \int_{-2}^{2} x^4\,dx$

Here, we assume, $y_x = x^4 = f(x)$

Since, the range $(-2,\ 2)$ consists of five equidistant ordinates, therefore, the range will be divided into four equal parts, i.e.,

$$2-(-2) = 4h \qquad [\because b-a = nh]$$

$$\Rightarrow \quad h = 1$$

Then, we have

$$y_{-2} = (-2)^4 = 16$$
$$y_{-1} = (-1)^4 = 1$$
$$y_0 = 0^4 = 0$$
$$y_1 = 1^4 = 1$$
$$y_2 = 2^4 = 16$$

(i) By Trapezoidal Rule, we get

$$\begin{aligned} \mathrm{I} &= h\left[\tfrac{1}{2}(y_{-2}+y_2) + (y_{-1}+y_0+y_1)\right] \\ &= 1.\left[\tfrac{1}{2}(16+16) + (1+0+1)\right] \\ &= 18 \end{aligned}$$

(ii) By Simpson's One-Third Rule, we get

$$\begin{aligned} \mathrm{I} &= \frac{h}{3}\left[(y_{-2}+y_2) + 4(y_{-1}+y_1) + 2y_0\right] \\ &= \frac{1}{3}\left[\,(16+16) + 4\,(1+1) + 2.0\,\right] \\ &= \frac{40}{3} \\ &= 13.33 \end{aligned}$$

(iii) By Simpson's Three-Eighth Rule, we get

$$\mathrm{I} = \frac{3h}{8}\left[(y_{-2}+y_2) + 3(y_{-1}+y_0) + 2y_1\right]$$

$= \frac{3}{8}\,[(16 + 16) + 3(1 + 0) + 2.1]$
$= \frac{3}{8}$ x 37
$= 13.875$

7. **Question :** Calculate by Simpson's One-Third Rule, an approximate value of $\int_1^5 \frac{dx}{x}$ when $h = 1$.

Solution : Suppose $\mathrm{I} = \int_1^5 \frac{dx}{x}$

Here, we assume, $y_x = \frac{1}{x} = f(x)$

Since, the range (1, 5) is divided into n equal parts so that $h = 1$.

∴ $5-1 = nh$ $[\because b - a = nh]$

⇨ $n = 4$

∴ The range (1, 5) consists of five equidistant ordinates and they are y_1, y_2, y_3, y_4, y_5.

∴ $y_1 = \frac{1}{1} = 1$

$y_2 = \frac{1}{2} = 0.5$

$y_3 = \frac{1}{3} = 0.33$

$y_4 = \frac{1}{4} = 0.25$

$y_5 = \frac{1}{5} = 0.20$

By Simpson's One-Third Rule, we get

$\mathrm{I} = \frac{h}{3}\,[(y_1 + y_5) + 4(y_2 + y_4) + 2y_3]$
$= \frac{1}{3}\,[\,(1+0.2) + 4\,(0.5+0.25) + 2 \text{ x } 0.33\,]$
$= \frac{1}{3}\,[\,1.2 + 3 + 0.66\,]$
$= 1.62$

8. **Question :** Calculate by Simpson's Three-Eighth Rule, an approximate value of $\int_0^6 \frac{dx}{1+x}$ by dividing the range into six equal parts.

Solution : Suppose $\mathrm{I} = \int_0^6 \frac{dx}{1+x}$

Here, we assume, $y_x = \frac{1}{1+x} = f(x)$

Since, the range (0, 6) is divided into 6 equal parts,

$\therefore \quad 6-0 = 6h \qquad [\because b - a = nh]$

$\Rightarrow \quad h = 1$

Since, the range (0, 6) is divided into 6 equal parts, therefore, the range (0, 6) consists of seven equidistant ordinates and they are $y_0, y_1, y_2, y_3, y_4, y_5, y_6$.

$$
\begin{aligned}
\therefore \quad y_0 &= \frac{1}{1} = 1 \\
y_1 &= \frac{1}{2} = 0.5 \\
y_2 &= \frac{1}{3} = 0.33 \\
y_3 &= \frac{1}{4} = 0.25 \\
y_4 &= \frac{1}{5} = 0.20 \\
y_5 &= \frac{1}{6} = 0.16667 \\
y_6 &= \frac{1}{7} = 0.14286
\end{aligned}
$$

By Simpson's Three-Eighth Rule, we get

$$
\begin{aligned}
I &= \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3] \\
&= \frac{3}{8}[(1+0.14286) + 3(0.5 + 0.33 + 0.20 + 0.16667) \\
&\quad + 2 \times 0.25] \\
&= \frac{3}{8}[1.14286 + 3 \times 1.19667 + 0.5] \\
&= \frac{3}{8} \times 5.23287 \\
&= 1.96233 \\
&= 1.96 \quad \text{approximately}
\end{aligned}
$$