Paper : MTHM – 601 (Statistics)
Measures of Dispersion

1. What is measures of dispersion ?
Solution : The measures that help us to know about the spread of a data set are called measures of dispersion.

Or, Dispersion is the measure of extent to which individual items vary.

Alternatively, suppose, the extent of spread or dispersion of the data are given in two cases i.e., case A and case B. And the dispersions are different in these cases. Then, the measurements of the scatter of the given data about the average (separately for both the cases) are said to be measures of dispersion or scatter.

2. What are the types of measures of dispersion ?
Solution : The types of the absolute measures of dispersion are given below :
(a) Range, (b) Quartile Deviation (Q.D.) or semi-interquartile range, (c) Mean deviation (MD) , (d) Standard deviation (SD)

3. What is Range ?
Solution : The difference between the largest (L) and smallest (S) values of a set of given data is known as Range.

Range = L – S

Co-efficient of Range = $\frac{L-S}{L+S}$

4. Calculate the range and co-efficient of range from the following data :
70, 75, 60, 22, 58, 80, 36

Solution :

Largest Value (L) = 80

Smallest Value (S) = 22

∴ Range = L – S = 80 – 22 = 58

Co-efficient of Range $= \frac{L-S}{L+S}$

$= \frac{80-22}{80+22}$

$= \frac{58}{102}$

$= 0.57$

5. From the following data calculate range and its co-efficient :

Class	:	10-20	20-30	30-40	40-50	50-60
Frequency	:	3	7	10	8	2

Solution : Range = L – S = 60 – 10 = 50

Co-efficient of Range $= \frac{L-S}{L+S}$

$= \frac{60-10}{60+10}$

$= \frac{50}{70}$

$= 0.71$

6. Write the Merits and Demerits of Range ?

Solution :

Merits :
(i) It is the simplest measure of dispersion.
(ii) It is easy to understand and calculate.
(iii) It is rigidly defined.

Demerits :
(i) It is not based on all observations. It is based on two extreme observations only.
(ii) It is affected much by fluctuations of sampling.
(iii) It is not capable of further algebraic treatment.

7. What is Quartile Deviation (Q.D.) ?

Solution : The difference between third and first quartile is known as interquartile range and half of this difference is known as semi-interquartile range or Q.D.

$$\text{Q.D.} = \frac{Q_3 - Q_1}{2}$$

$$\text{Co-efficient of Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

8. Calculate the Q.D. and the co-efficient of Q.D. from the following data :

Class :	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency :	10	15	25	40	35	20	5

<u>Solution</u> : From the given data, we prepare a table as :

Class	Frequency (f)	Cumulative Frequency (F)
5 – 10	10	10
10 – 15	15	25
15 – 20	25	50
20 – 25	40	90
25 – 30	35	125
30 – 35	20	145
35 – 40	5	150
	N = 150	

Here, N = 150, $\frac{N}{4} = 37.5$, $\frac{3N}{4} = 112.5$, h = 5 (h is the interval of the class)

Since, c.f. just greater than 37.5 is 50. Hence, the corresponding class 15 – 20 is the Q_1 class.

$$Q_1 = L + \frac{\frac{N}{4} - F}{f}\, h$$

$$= 15 + \frac{37.5 - 25}{25} \text{ x } 5 = 15 + 2.5 = 17.5$$

Since, c.f. just greater than 112.5 is 125. Hence, the corresponding class 25 – 30 is the Q_3 class.

$$Q_3 = L + \frac{\frac{3N}{4} - F}{f}\, h$$

$$= 25 + \frac{112.5 - 90}{35} \text{ x } 5 = 25 + 3.2 = 28.21$$

$\therefore$ Q.D. $= \frac{Q_3 - Q_1}{2} = \frac{28.21 - 17.5}{2} = 5.4$

Co-efficient of Q.D. $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{28.21 - 17.5}{28.21 + 17.5} = \frac{10.7}{45.7} = 0.23$

9. Write the Merits and Demerits of Range ?

<u>Solution</u> :

Merits : (i) It is easy to understand and calculate.
(ii) It is not affected by extreme values.

(iii) It is rigidly defined.

Demerits : (i) It is based on 50% observations only.
(ii) It is not suitable for further algebraic treatment.
(iii) It is affected much by fluctuations of sampling.

10. What is Mean deviation ?

Solution : The mean deviation is the sum of the absolute values of the deviations from the mean divided by the number of items, (i.e., the sum of the frequencies).

For ungrouped data : Suppose, $x_1, x_2, x_3, \ldots\ldots, x_n$ are values of a variable, then

MD = $\frac{1}{n}\sum|x_i - A|$ where 'A' may be Mean, Median or Mode

For grouped data : Suppose, $x_1, x_2, x_3, \ldots\ldots, x_n$ are values of a variable with frequencies $f_1, f_2, f_3, \ldots\ldots., f_n$, then

MD = $\frac{1}{n}\sum f_i|x_i - A|$ where 'A' may be Mean, Median or Mode

Co-efficient of MD = $\frac{MD}{A}$

11. The following are the marks of 7 students in Mathematics. Find

(i) MD from mean and its co-efficient.

(ii) MD from median and its co-efficient.

18, 26, 15, 20, 17, 12, 25

Solution : (i)

x	$x - \bar{x}$	$\lvert x - \bar{x}\rvert$
18	–1	1
26	7	7
15	–4	4
20	1	1
17	–2	2
12	–7	7
25	6	6
$\sum x = 133$		$\sum\lvert x - \bar{x}\rvert = 23$

$$\bar{x} = \frac{\sum x}{n} = \frac{133}{7} = 19$$

$$\therefore MD \text{ from Mean} = \frac{\sum|x-\bar{x}|}{n} = \frac{28}{7} = 4$$

$$\text{Co-efficient of } MD \text{ from Mean} = \frac{MD}{Mean} = \frac{4}{19} = 0.21$$

(ii) Arranging the marks in ascending order, we get

12, 15, 17, 18, 20, 25, 26 (18 is the median)

x	x-Meadian	$\lvert x - Median\rvert$
18	0	0
26	8	8
15	–3	3
20	2	2
17	–1	1
12	–6	6
25	7	7
		$\sum\lvert x - Median\rvert = 27$

$$\therefore MD \text{ from Median} = \frac{\sum|x-Median|}{n} = \frac{27}{7} = 3.9$$

$$\text{Co-efficient of } MD \text{ from Median} = \frac{MD}{Median} = \frac{3.9}{18} = 0.22$$

12. Find MD from Mean from the following discrete distribution.

Items :	5	15	25	35	45
Frequency :	5	8	15	16	6

Solution :

x	f	xf	$x - \bar{x}$	$\lvert x - \bar{x}\rvert$	$f\lvert x - \bar{x}\rvert$
5	5	25	–22	22	110
15	8	120	–12	12	96
25	15	375	–2	2	30
35	16	560	8	8	128
45	6	270	18	18	108
	N = 50	$\sum xf = 1350$			$\sum f\lvert x - \bar{x}\rvert = 472$

$$\bar{x} = \frac{\sum xf}{N} = \frac{1350}{50} = 27$$

$\therefore MD$ from Mean = $\frac{\sum|x-\bar{x}|}{N} = \frac{472}{50}$ = 9.44

13. Find MD from Mean from the following frequency distribution.

Items :	10-20	20-30	30-40	40-50	50-60
Frequency :	7	10	20	10	3

Solution :

x	f	x	xf	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
10-20	7	15	105	18.4	128.8
20-30	10	25	250	8.4	84
30-40	20	35	700	1.6	32
40-50	10	45	450	11.6	116
50-60	3	55	165	21.6	64.8
	N = 50		$\sum xf = 1670$		$\sum f\lvert x-\bar{x}\rvert = 425.6$

$$\bar{x} = \frac{\sum xf}{N} = \frac{1670}{50} = 33.4$$

$\therefore MD$ from Mean = $\frac{\sum|x-\bar{x}|}{N} = \frac{425.6}{50}$ = 8.5

14. Write the Merits and Demerits of Range ?

Solution : Merits : (i) It is easy to understand and calculate.
(ii) It is based on all observations.
(iii) It is not very much affected by extreme values.
(iv) It is rigidly defined.

Demerits : (i) Mean deviation ignores the algebraic signs of the deviations and hence it is not capable of further algebraic treatment.
(ii) It is not an accurate measure, particularly when it is calculated from mode.

Note : (i) Mean deviation is least when it is measured from mean.
(ii) Mean deviation about mean is independent of the change in origin but not of scale.

15. What is Standard Deviation (SD) ?

Solution : SD of set of values is the positive square root of the A.M. of squared deviation when deviations are taken from mean. It is denoted by $\boldsymbol{\sigma}$. The square of SD is called variance. Thus, SD = $\sqrt{variance}$.

For ungrouped data : Suppose, $x_1, x_2, x_3, \ldots\ldots, x_n$ are values of a variable x, then

$$\boldsymbol{\sigma} = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2} = \sqrt{\frac{1}{n}\sum x_i^2 - \bar{x}^2}$$

For grouped data : Suppose, $x_1, x_2, x_3, \ldots\ldots, x_n$ are values of a variable x with respective frequencies $f_1, f_2, f_3, \ldots\ldots., f_n$, then

$$\boldsymbol{\sigma} = \sqrt{\frac{1}{N}\sum f_i(x_i - \bar{x})^2} = \sqrt{\frac{1}{N}\sum f_i x_i^2 - \bar{x}^2}$$

Short cut method for calculating SD : SD can also calculated by this method

(i) For ungrouped data : $\boldsymbol{\sigma} = \sqrt{\frac{1}{n}\sum d^2 - \left(\frac{1}{n}\sum d\right)^2}$ where d = x – A,

A = Assumed mean.

(ii) For grouped data : $\boldsymbol{\sigma} = \sqrt{\frac{1}{N}\sum fd^2 - \left(\frac{1}{N}\sum fd\right)^2}$ where d = x – A,

A = Assumed mean.

(iii) For continuous data with equal class interval :

$$\boldsymbol{\sigma} = \sqrt{\frac{1}{N}\sum fd^2 - \left(\frac{1}{N}\sum fd\right)^2} \times h \qquad \text{where } d = \frac{x-A}{h}$$