

## Second Order ODE with RHM is a function of $x$ ( Trigonometric function )

1.  $(D^2 - 4)y = \sin 2x$

Solution : Given equation is

$$(D^2 - 4)y = \sin 2x \dots \text{(i)}$$

The auxiliary equation of (i) will be

$$m^2 - 4 = 0$$

$$\Rightarrow m = \pm 2$$

$$\therefore C.F. = c_1 e^{2x} + c_2 e^{-2x}$$

$$\text{And, } P.I. = \frac{1}{D^2 - 4} \sin 2x$$

$$= \frac{\sin 2x}{-2^2 - 4}$$

$$= -\frac{1}{8} \sin 2x$$

$\therefore$  The general solution,  $y = C.F. + P.I.$

$$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{8} \sin 2x \quad \text{Answer.}$$

2.  $(D^2 + 4)y = \sin 2x$

Solution : Given equation is

$$(D^2 + 4)y = \sin 2x \dots \text{(i)}$$

The auxiliary equation of (i) will be

$$m^2 + 4 = 0$$

$$\Rightarrow m = \pm 2i$$

$$\therefore C.F. = A \cos 2x + B \sin 2x$$

$$\text{And, } P.I. = \frac{1}{D^2+4} \sin 2x \dots \text{(ii)}$$

$$\text{Let us consider, } Y = \frac{1}{D^2+4} \cos 2x, \quad Z = \frac{1}{D^2+4} \sin 2x$$

$$\begin{aligned}\therefore Y + iZ &= \frac{1}{D^2+4} \cos 2x + i \frac{1}{D^2+4} \sin 2x \\&= \frac{1}{D^2+4} (\cos 2x + i \sin 2x) \\&= \frac{1}{D^2+4} e^{i2x} \quad [\because \cos \theta + i \sin \theta = e^{i\theta}] \\&= e^{i2x} \frac{1}{(D+2i)^2+4} 1 \\&= e^{i2x} \frac{1}{D^2+4iD+4i^2+4} 1 \\&= e^{i2x} \frac{1}{4iD+D^2} 1 \\&= e^{i2x} \cdot \frac{1}{4iD \left(1 + \frac{D}{4i}\right)} 1 \\&= e^{i2x} \cdot \frac{1}{4iD} \left(1 + \frac{D}{4i}\right)^{-1} 1 \\&= e^{i2x} \cdot \frac{1}{4iD} \left\{1 - \frac{D}{4i} + \left(\frac{D}{4i}\right)^2 - \dots\right\} 1 \\&= e^{i2x} \cdot \frac{1}{4i} \cdot \frac{1}{D} \cdot 1 \\&= e^{i2x} \cdot \frac{1}{4i} \cdot \chi \\&= \frac{x}{4i} (\cos 2x + i \sin 2x) \\&= -\frac{ix \cos 2x}{4} + \frac{x \sin 2x}{4}\end{aligned}$$

$$Y + iZ = \frac{x \sin 2x}{4} + i \left( \frac{-x \cos 2x}{4} \right)$$

Equating real and imaginary parts on both sides, we get

$$Y = \frac{x \sin 2x}{4} \text{ and } Z = \frac{-x \cos 2x}{4}$$

$$\therefore P.I. = \frac{-x \cos 2x}{4}$$

*∴ The general solution,  $y = C.F. + P.I.$*

$$y = A \cos 2x + B \sin 2x - \frac{x \cos 2x}{4} \quad \text{Answer.}$$

$$3. \frac{d^2y}{dx^2} + y = \sin x$$

Solution : Given equation is

$$\frac{d^2y}{dx^2} + y = \sin x$$

$$\Rightarrow (D^2 + 1)y = \sin x \dots \dots \dots \text{(i)}$$

The auxiliary equation of (i) will be

$$m^2 + 1 = 0$$

$$\Rightarrow m = \pm i$$

$$\therefore C.F. = A \cos x + B \sin x$$

$$\text{And P.I.} = \frac{1}{D^2+1} \sin x$$

$$\text{Let us take, } Y = \frac{1}{D^2+1} \cos x, \quad Z = \frac{1}{D^2+1} \sin x$$

$$\therefore Y + iZ = \frac{1}{D^2+1} \cos x + i \frac{1}{D^2+1} \sin x$$

$$= \frac{1}{D^2+1} (\cos x + i \sin x)$$

$$\begin{aligned}
&= \frac{1}{D^2+1} e^{ix} \\
&= e^{ix} \frac{1}{(D+i)^2+1} 1 \\
&= e^{ix} \frac{1}{D^2+2iD+i^2+1} 1 \\
&= e^{ix} \frac{1}{2iD+D^2} 1 \\
&= e^{ix} \frac{1}{2iD\left(1+\frac{D}{2i}\right)} 1 \\
&= e^{ix} \cdot \frac{1}{2iD} \left(1 + \frac{D}{2i}\right)^{-1} 1 \\
&= e^{ix} \cdot \frac{1}{2iD} \left\{1 - \frac{D}{2i} + \left(\frac{D}{2i}\right)^2 - \dots\right\} 1 \\
&= e^{ix} \cdot \frac{1}{2iD} 1 \\
&= e^{ix} \cdot \frac{1}{2i} \cdot \frac{1}{D} 1 \\
&= e^{ix} \cdot \frac{1}{2i} \cdot x \\
&= \frac{xe^{ix}}{2i} \\
&= \frac{x}{2i} (\cos x + i \sin x) \\
&= -\frac{ix \cos x}{2} + \frac{x \sin x}{2} \\
\therefore Y + iZ &= \frac{x \sin x}{2} + i \left(-\frac{x \cos x}{2}\right)
\end{aligned}$$

Equating real and imaginary parts, we get

$$\therefore Y = \frac{x \sin x}{2} \text{ and } Z = -\frac{x \cos x}{2}$$

$$\therefore \text{P.I.} = -\frac{x \cos x}{2}$$

*∴ The general solution,  $y = C.F. + P.I.$*

$$y = A \cos x + B \sin x - \frac{x \cos x}{2} \quad \text{Answer.}$$

$$4. (D^2 - 5D + 6)y = \sin 3x$$

Solution : Given equation is

$$(D^2 - 5D + 6)y = \sin 3x \dots \dots \dots \text{(i)}$$

The auxiliary equation of (i) will be

$$m^2 - 5m + 6 = 0$$

$$\Rightarrow (m - 2)(m - 3) = 0$$

$$\Rightarrow m = 2, 3$$

$$\therefore C.F. = c_1 e^{2x} + c_2 e^{3x}$$

$$\text{And, } P.I. = \frac{1}{D^2 - 5D + 6} \sin 3x$$

$$= \frac{1}{-3^2 - 5D + 6} \sin 3x$$

$$= \frac{1}{-3 - 5D} \sin 3x$$

$$= \frac{-1}{5D + 3} \sin 3x$$

$$= \frac{-(5D - 3)}{(5D + 3)(5D - 3)} \sin 3x$$

$$= \frac{-(5D - 3)}{25D^2 - 9} \sin 3x$$

$$= \frac{-(5D - 3)}{25(-3^2) - 9} \sin 3x$$

$$= \frac{-(5D - 3)}{-234} \sin 3x$$

$$= \frac{5D \sin 3x}{234} - \frac{3 \sin 3x}{234}$$

$$= \frac{1}{234} (5.3\sin 3x - 3\sin 3x)$$

$$= \frac{1}{78} (5\cos 3x - \sin 3x)$$

$\therefore$  The general solution,  $y = C.F. + P.I.$

$$y = c_1 e^{2x} + c_2 e^{3x} + \frac{1}{78} (5\cos 3x - \sin 3x) \quad \text{Answer.}$$